MATH 2020 Advanced Calculus II

Tutorial 1

Compute the following double integrals:

1. $\int_0^1 \int_0^1 \frac{x}{1+xy} dxdy$

   Solution.

$$\begin{aligned}
\int_0^1 \int_0^1 \frac{x}{1+xy} dxdy &= \int_0^1 \int_0^1 \frac{x}{1+xy} dydx \\
&= \int_0^1 \left[\ln(1+xy)\right]_0^1 dx \\
&= \int_0^1 \left[\ln(1+x) - \ln(1+0)\right] dx \\
&= \left[(1+x)\ln(1+x)\right]_0^1 - \int_0^1 dx \\
&= 2\ln 2 - 1
\end{aligned}$$

2. $\int_0^2 \int_0^1 xy^2 e^{xy^3} dxdy$

   Solution.

$$\begin{aligned}
\int_0^2 \int_0^1 xy^2 e^{xy^3} dxdy &= \int_0^2 \int_0^1 xy^2 e^{xy^3} dydx \\
&= \int_0^2 \left[\frac{1}{3} e^{xy^3}\right]_0^1 dx \\
&= \frac{1}{3} \int_0^2 (e^x - 1) dx \\
&= \frac{1}{3} \left[e^x - x\right]_0^2 \\
&= \frac{1}{3}(e^2 - 3)
\end{aligned}$$

3. $\int_1^2 \int_1^2 \frac{1}{x^2 y^3} dxdy$

   Solution.

$$\begin{aligned}
\int_1^2 \int_1^2 \frac{1}{x^2 y^3} dxdy &= \left(\int_1^2 \frac{dx}{x^2}\right)\left(\int_1^2 \frac{dy}{y^3}\right) \\
&= \left[-\frac{1}{x}\right]_1^2 \left[-\frac{1}{2y^2}\right]_1^2 \\
&= \left(-\frac{1}{2} + 1\right)\left(-\frac{1}{8} + \frac{1}{2}\right) \\
&= \frac{1}{2} \times \frac{3}{8} \\
&= \frac{3}{16}
\end{aligned}$$

4. $\int_{-1}^{1}\int_{-2}^{2}(x^2+y^2)dxdy$

Solution.

$$\begin{aligned}\int_{-1}^{1}\int_{-2}^{2}(x^2+y^2)dxdy &= \int_{-1}^{1}\left[\frac{x^3}{3}+xy^2\right]_{-2}^{2}dy\\ &= \int_{-1}^{1}\left(\frac{16}{3}+4y^2\right)dy\\ &= \left[\frac{16}{3}y+\frac{4}{3}y^3\right]_{-1}^{1}\\ &= \frac{32}{3}+\frac{8}{3}\\ &= \frac{40}{3}\end{aligned}$$

5. $\int_{0}^{1}\int_{0}^{1}(1-x-y)dxdy$

Solution.

$$\begin{aligned}\int_{0}^{1}\int_{0}^{1}(1-x-y)dxdy &= \int_{0}^{1}\left[x-\frac{x^2}{2}-xy\right]_{0}^{1}dy\\ &= \int_{0}^{1}\left(1-\frac{1}{2}-y\right)dy\\ &= \int_{0}^{1}\left(\frac{1}{2}-y\right)dy\\ &= \frac{1}{2}-\frac{1}{2}\\ &= 0\end{aligned}$$